On the Optimal Reactive Power Control for Grid-Connected Photovoltaic Distributed Generation Systems

Manel Velasco, Pau Martí
Automatic Control Department
Technical University of Catalonia
Pau Gargallo 5, 08028 Barcelona, Spain
Email: manel.velasco@upc.edu

Javier Torres-Martínez, Jaume Miret and Miguel Castilla
Department of Electronic Engineering
Technical University of Catalonia
Av. Víctor Balaguer s/n, 08800 Vilanova i la Geltrú, Spain
Email: miquel.castilla@upc.edu

Abstract—The increasing deployment of distributed generation (DG) such as photovoltaic panels (PV) connected to low-voltage (LV) grids is becoming a common trend in urban areas. The advances of information and communication technology (ICT) facilitates the collaborative operation of DG systems to achieve collective benefits. The fusion of these two trends creates a new scenario where reactive power control methods can offer additional features and benefits beyond the conventional voltage regulation provided by the droop method. Taking advantage of this new scenario, this paper formulates the application of reactive power control as an optimization problem where simple and ideal settings are imposed by design in order to facilitate the exploration search as well as to avoid over-constraining the optimization space. By appropriately using the power capacity of inverters, the desired collective benefit is to minimize power losses while individual voltages at each inverter should be kept within the statutory limits. The simulated solution of the optimization problem is applied to a real-inspired PV-LV grid subject to an over-voltage situation, which may typically occur during periods of high production but low consumption. Simulation results reveal the optimal settings for reactive power control set-points at each inverter, which calls for a final discussion to review the applicability of the optimization approach.

I. INTRODUCTION

The increasing deployment of distributed generation (DG) such as photovoltaic panels (PV) in grid-connected low-voltage (LV) grids is becoming a common trend in urban areas. Although this trend has been approached in the literature from different angles but primarily taking into account technical [1] and economic [2] perspectives, widespread implementation of the diverse available methods has not taken place [3], [4]. In a parallel track, the characteristics of actively managed networks [5]–[7], where control schemes employing real-time control and communication systems allow more effective management of different network participants, including DG plants, storage and demand, permits to design novel collaborative strategies to achieve collective benefits.

These two trends create new scenarios where reactive power control methods can still mitigate some of the problems introduced by DG [8] while offering additional features and benefits beyond the conventional voltage regulation provided by the droop method. Looking at existing challenges, for example, for grid-connected photovoltaic systems, when low power demand coincides with high solar radiation, voltage rise may occur [9], i.e. grid voltage can be outside of the statutory limits specified by the grid codes [10]. The over-voltage problem calls for a management scheme able to alleviate the excessive voltage increase by means of properly integrating real-time control and possibly information and communication technology [11], [12]. Regarding the possible use of ICT, simple voltage management methods use only local measurements and do not require data transfer between DGs, e.g. [13]–[15]. However, such decentralized methods may evolve to distributed or centralized approaches when ICT is applied, and local actions may be based on both local and global measurements, e.g. [16]–[23].

Within the existing control techniques to deal with the voltage rise problem, reactive power control methods have been proved to successfully bring DGs voltages within the admissible voltage range without reducing the active power of DGs. In residential neighborhoods with high PV penetration [24], PV inverters can be used for reactive power supply/absorption when the real power injection is less than the inverter rated power.

Taking into account the capability of regulating voltage through appropriate reactive power settings often dictated by a local droop-like method at each inverter, this paper presents a study that looks for the optimal reactive power settings that minimize power losses (in lines and inverters). The starting point is an ideal PV-LV grid where inverters can inject/absorb any quantity of reactive power $Q$ regardless of each individual injected active power $P$. That is, the power factor is not considered a constraint.

In order to limit the optimization space, the reactive power at each inverter is determined by a simple expression of the diverse existing droop curves [15]. The considered droop is a straight line given in the slope-intercept form. The slope and the y-intercept are the free parameters that can be tuned to provide optimal settings for $Q$ at each inverter. And the initial scenario for searching the optimal $Q$ values is an over voltage situation in a PV-LV grid. The problem of optimizing grid
parameters, size, locations, etc with respect to energy losses has been previously treated, e.g. [25]–[28]. The approach presented in this paper is a complementary study that gives an intuitive and simple optimization procedure that applies to an ideal system without technological constraints (in the power systems itself or in the possible demands of ICT). The simplicity helps at providing valuable insight for future design of collaborative operation of distributed generation systems regulated by reactive power control methods.

The rest of this paper is structured as follows. Section II presents the photovoltaic grid under consideration. Section III presents the optimization procedure for reactive power control. Section IV presents the simulated results. And Section V concludes the paper.

II. GRID MODEL

The PV-LV grid under study and illustrated in Figure 1 is based on real data from a residential area located in Brædstrup, a village in the region of Østjylland, Denmark [15]. The MV grid (including the MV to LV transformer) is modeled as an ideal source with amplitude 230 Vrms and frequency 50 Hz which is connected to the node 0 of the installation. After, a configurable topology with 24 nodes is built. The topology can be considered a mesh topology when the two breakers, B1 and B2, are closed. This would be the standard topology to interconnect the source, breakers, and nodes. The single-line

impedance values of these cables are listed in Table I.

Figure 2 shows the PV generators and the local loads included in the node j of the LV grid (for j = 1, . . . , 24). The figure also draws the line impedances between the nodes i − j and j − k, which are modeled as a series connection of a resistance and an inductance, R_{i−j}L_{i−j}, with 0.32 · d_{i} Ω and (0.082 · d_{i})/(2 · π · 50) H, where d_{i} stands for the distance between nodes (either 0.035, 0.070 or 0.105Km in the general case). For simplicity, this study considers constant inductive loads and it also considers that all the PV generators have the same power production (P_{i} = 18 kW) (even knowing that solar modules have different size and a non-constant active power generation).

In the presented LV grid, the over voltage problem is intentionally caused by the power production settings of the PV generators. In fact the power production has been specified such that in the case of having the two breakers in closed state only a few generators have their local voltage slightly above 1.1 p.u. (per unit), as shown in the top sub-figure of Figure 3. The bottom sub-figure shows the unusual case of having breaker B2 in open state. In this case the majority of local voltages are above 1.1 p.u.

Since the impedance of the interconnecting lines is mainly resistive, the injection of active power to the grid notably increases the local voltage at each node. The worst case scenario corresponds to a situation characterized by the maximum power production and minimum power consumption. This problem is typically overcome thanks to the capacity of the PV inverters to absorb reactive power.

III. OPTIMAL REACTIVE POWER CONTROL

Loosely speaking, many reactive power control policies designed to cope with voltage rise imply determining which

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Line} & \text{Nodes} & \text{Impedance (mΩ)} & \text{Observation} \\
\hline
1 & 0 - 1 & 22.4 + j 5.7 & \text{Line containing breaker B1} \\
2 & 0 -13 & 67.2 + j 17.2 & \text{Line containing breaker B2} \\
3 & 1 - 2 & 11.2 + j 2.9 \\
4 & 1 - 4 & 22.4 + j 5.7 \\
5 & 2 - 3 & 11.2 + j 2.9 \\
6 & 4 - 5 & 11.2 + j 2.9 \\
7 & 4 - 7 & 22.4 + j 5.7 \\
8 & 5 - 6 & 11.2 + j 2.9 \\
9 & 7 - 8 & 11.2 + j 2.9 \\
10 & 7 -13 & 33.6 + j 8.6 \\
11 & 8 - 9 & 11.2 + j 2.9 \\
12 & 9 -10 & 11.2 + j 2.9 \\
13 & 10 -11 & 11.2 + j 2.9 \\
14 & 11-12 & 11.2 + j 2.9 \\
15 & 13 -14 & 22.4 + j 5.7 \\
16 & 13 - 16 & 22.4 + j 5.7 \\
17 & 14 - 15 & 11.2 + j 2.9 \\
18 & 16 - 17 & 22.4 + j 5.7 \\
19 & 16 - 20 & 22.4 + j 5.7 \\
20 & 17 - 18 & 11.2 + j 2.9 \\
21 & 18 - 19 & 11.2 + j 2.9 \\
22 & 20 - 21 & 11.2 + j 2.9 \\
23 & 20 - 22 & 11.2 + j 2.9 \\
24 & 20 - 23 & 22.4 + j 5.7 \\
25 & 23 - 24 & 11.2 + j 2.9 \\
\hline
\end{array}
\]

\[\text{Table I: Line impedance values of the underground cables.}\]
reactive power set-points have to be given to each inverter in order to keep the line voltage within the specified range. And this achieved locally by different sorts of the droop control approaches.

A. Revisiting Droop Approaches

The simplest droop approach may be to specify fixed droop settings for generating reactive power reference values in terms of either local voltage (\(V\)) or injected real power (\(P\)). The static droop curves are supposed to be configured identically for all inverters which are located under the same transformer. Focusing on the \(V-Q\) droop curve, which is the one considered in this work because it has superior performance on grid dynamics. The simplest form for optimization purposes of the \(V-Q\) droop curve, which is the one considered in (1), can be found. For example, instead of applying the same droop slope \(m_i\) for all inverters which are located under the same transformer.

\[
J_{\text{losses}} = \sum_{j=1}^{n} \left( R_{G_j} I_{G_j}^2 + R_{L_{j-1}} I_{L_{j-1}}^2 \right) \tag{4}
\]

where \(R_{G_j}\) is the loss at the \(j\)th-inverter, \(R_{L_{j-1}} I_{L_{j-1}}^2\) is the line loss in each branch between consecutive inverters, and \(n\) is the inverter number.

Therefore, considering the updated droop function (3) and the objective function (4), the optimization problem can be formulated as follows

\[
\min_{Q_i^*} J_{\text{losses}} \tag{5}
\]

with respect to \(Q_i^*\)

subject to \(V_i \leq 1.1\)

B. Analyzing the Droop Curve

The simplest form for optimization purposes of the \(V-Q\) droop (1) is given by

\[
y = ax + b \tag{2}
\]

The slope-intercept form provides two optimization parameters, \(a\) and \(b\). The optimization procedure could explore both parameters. However, as discussed next, we will limit the exploration over the intercept \(b\) parameter (one \(b\) per node, that is, \(24\) \(b\)’s).

Looking at the slope \(a\) in (2) which corresponds to the droop slope \(m_i\) in (1), changing its value will definitely permit changing the \(Q\) set-point for each node: the bigger the slope, the higher the \(Q\) set-point for each node. However, when the slope changes sign, which occurs in the vertical position of the straight line, that is when \(a = \pm \infty\), and the desired injection of \(Q\) changes to desired absorption, the search of the optimization procedure may become unstable. This suggests avoiding the use of the droop slope parameter as an optimization variable in a first optimization study. However, looking at the \(y\)-intercept \(b\) in (2), the search instability problem does not appear since the shifting in the \(y\)-axis of the curve given by \(b\) permits a smooth transition between absorbing and injecting reactive power \(Q\).

Therefore, the droop function (1) used for the optimization procedure is updated to

\[
Q_i = m_i(V_i - 1) + Q_i^* \tag{3}
\]

where \(Q_i^*\) will be the optimization variable.

C. Formulation of the Optimization Problem

The collective goal of the DG system is to minimize energy losses, including power losses in the underground cables and in the PV inverters. To this extend, the objective function can be written as

\[
J_{\text{losses}} = \sum_{j=1}^{n} \left( R_{G_j} I_{G_j}^2 + R_{L_{j-1}} I_{L_{j-1}}^2 \right) \tag{4}
\]

where \(R_{G_j}\) is the loss at the \(j\)th-inverter, \(R_{L_{j-1}} I_{L_{j-1}}^2\) is the line loss in each branch between consecutive inverters, and \(n\) is the inverter number.

Therefore, considering the updated droop function (3) and the objective function (4), the optimization problem can be formulated as follows

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\]

with respect to \(Q_i^*\)

subject to \(V_i \leq 1.1\)

D. Simulation approach of the optimization

The implementation of the PV-LV grid has been performed using Matlab/Simulink. The optimization problem (5) is solved using the \texttt{fminsearch} in a recursive algorithm given an initial value to all \(Q_i\). In addition, to avoid abrupt changes in the control settings of the LV grid regarding \(Q_i^*\), rather than directly imposing the possibly optimal setting, this value renamed as \(Q_i^o\) is given to a proportional-integral (PI) controller as in

\[
Q_i^o = K_p (Q_i^o - Q_m) + K_i \int (Q_i^o - Q_m) dt \tag{6}
\]

The effect is to drive the local measured reactive power \(Q_m\) to the optimal setting indicated by \(Q_i^*\) with a controlled transient dynamics.
shown in the middle sub-figure.

For illustrative purposes, each inverter initial optimal set-point of the two breakers in closed state. The intermediate optimal reactive setting $Q^*_{in}$ for the problem (5) have been obtained with an initial condition of $Q^*_{in} = 18000 \text{VAR}$ for each inverter, after running an arbitrary number of iterations of the \texttt{fminsearch} recursive algorithm.

The top sub-figure of Figure 5 shows the intermediate stationary voltages achieved by the optimal reactive power settings. As it can be seen, the local voltages lie within the statutory limits (to be compared with the top sub-figure of Figure 3). It is interesting to observe that more than one terminal voltage goes exactly at 1.1 p.u. This property can not be usually achieved by existing conventional droop approaches. However, the optimization procedure can do it. Moreover, although the local voltages already meet the control specifications, the \texttt{fminsearch} algorithm still has room to iterate to achieve a better cost for (4), as it will be described in next subsection.

The bottom sub-figure of Figure 5 shows the intermediate optimal reactive settings given by the droop equation (3) where the y-intercept term $Q^*_{in}$ has been optimally found using the simulation approach explained in subsection III-D that solves the problem posed in subsection III-C for an arbitrary number of iterations. The reactive profile indicates that the power losses are minimized by a particular (and difficult to predict) specification of reactive power injection (positive $Q$) and absorption (negative $Q$). This intermediate result may be not intuitive. In addition, the power factor that would imply such injection/absorption of reactive power given the injected active power may be outside of the standard and permitted inverters operation. The middle sub-figure of Figure 5 shows the stationary currents that are generated with the intermediate optimal settings for $Q^*_{in}$.

Although the figures corresponding to the case of having breaker B2 in open state have been omitted due to space limitations, the same type of intermediate optimal settings are achieved. That is, the intermediate reactive power profile has a non-predictive/reasonable shape.

### C. Final Optimal Reactive Power Settings

Figure 6 shows the same results as before, that is, voltages (top sub-figure), currents (middle sub-figure), and optimal injected/absorbed reactive power (bottom sub-figure) at each inverter when the \texttt{fminsearch} recursive algorithm settles to an optimal value for the cost (4) and no significant improvement can be further achieved, for the scenario with breakers B1 and B2 closed (the results obtained for B2 open go into the same direction).

Comparing the intermediate results shown in Figure 5 with the final results shown in Figure 6, the following observations can be stated. First of all, the voltage profile in both cases (top sub-figures) is almost the same. This corroborates the idea that beyond simply regulating voltage, collective benefits

\[ V_{	ext{stat}}(\text{p.u.}) = 1.1 \frac{V_{	ext{nom}}}{V_{	ext{ref}}} \]

\[ I_{	ext{stat}}(\text{Arms}) = 50 \]

\[ \text{React. Power (VAR)} = 10 \]

\[ \text{Time (s)} = 15 \]

**IV. SIMULATION RESULTS**

This section presents the simulation results and discuss several aspects of the presented study.

#### A. Transient Dynamics

Before describing the main results, Figure 4 shows the transient dynamics of the simulated grid where active power injection begins at $t = 1 \text{ s}$ and the reactive power control starts at $t = 2 \text{ s}$. The three sub-figures correspond to the case of the 24 PV-LV grid presented in Section II for the case of the two breakers in closed state, which would be the usual operation. For illustrative purposes, each inverter initial optimal set-point obeys that $Q^*_{in} = -1600 \cdot n \text{VAR}$ where $n = 1, \ldots, 24$ denotes each of the nodes in Figure 1.

As it can be seen in the top sub-figure, the local voltages smoothly settle to values below 1.1pu while the reactive power absorbed by each inverter shown in the bottom sub-figure reaches the given set-point. And this generates the currents shown in the middle sub-figure.

The setting of $Q^*_{in} = -1600 \cdot n \text{VAR}$ for each inverter could have been an initializing operating point from which to look for the optimal $Q^*_{in}$ values. However, it has been only used here to easier illustrate the dynamics in all nodes with different reactive power values.

#### B. Intermediate Optimal Reactive Power Settings

Figure 5 shows intermediate optimal reactive power settings. The three sub-figures correspond to the same case as before, that is, the 24 PV-LV grid presented in Section II for the case of the two breakers in closed state. The intermediate optimal reactive setting $Q^*_{in}$ for the problem (5) have been obtained with an initial condition of $Q^*_{in} = 18000 \text{VAR}$ for each inverter, after running an arbitrary number of iterations of the \texttt{fminsearch} recursive algorithm.

The top sub-figure of Figure 5 shows the intermediate stationary voltages achieved by the optimal reactive power settings. As it can be seen, the local voltages lie within the statutory limits (to be compared with the top sub-figure of Figure 3). It is interesting to observe that more than one terminal voltage goes exactly at 1.1 p.u. This property can not be usually achieved by existing conventional droop approaches. However, the optimization procedure can do it. Moreover, although the local voltages already meet the control specifications, the \texttt{fminsearch} algorithm still has room to iterate to achieve a better cost for (4), as it will be described in next subsection.

The bottom sub-figure of Figure 5 shows the intermediate optimal reactive settings given by the droop equation (3) where the y-intercept term $Q^*_{in}$ has been optimally found using the simulation approach explained in subsection III-D that solves the problem posed in subsection III-C for an arbitrary number of iterations. The reactive profile indicates that the power losses are minimized by a particular (and difficult to predict) specification of reactive power injection (positive $Q$) and absorption (negative $Q$). This intermediate result may be not intuitive. In addition, the power factor that would imply such injection/absorption of reactive power given the injected active power may be outside of the standard and permitted inverters operation. The middle sub-figure of Figure 5 shows the stationary currents that are generated with the intermediate optimal settings for $Q^*_{in}$.

Although the figures corresponding to the case of having breaker B2 in open state have been omitted due to space limitations, the same type of intermediate optimal settings are achieved. That is, the intermediate reactive power profile has a non-predictive/reasonable shape.

#### C. Final Optimal Reactive Power Settings

Figure 6 shows the same results as before, that is, voltages (top sub-figure), currents (middle sub-figure), and optimal injected/absorbed reactive power (bottom sub-figure) at each inverter when the \texttt{fminsearch} recursive algorithm settles to an optimal value for the cost (4) and no significant improvement can be further achieved, for the scenario with breakers B1 and B2 closed (the results obtained for B2 open go into the same direction).

Comparing the intermediate results shown in Figure 5 with the final results shown in Figure 6, the following observations can be stated. First of all, the voltage profile in both cases (top sub-figures) is almost the same. This corroborates the idea that beyond simply regulating voltage, collective benefits
can be achieved by an appropriate use of the power capacity of each inverter.

Second, by comparing the bottom sub-figures that provide the reactive settings $Q_i^\ast$, the final reactive profile (bottom sub-figure of Figure 6) exhibits a more "linear" pattern than the intermediate profile (bottom sub-figure of Figure 5). In fact, the obtained final profile roughly indicates that almost all inverters "do nothing" (the injected/absorbed reactive power is around zero) except for the last two inverters (nodes 23 and 24 in Figure 1) that are in charge of absorbing the particular amount of reactive power that keeps all inverters voltages within the statutory limits while power losses are minimized.

Third, it is interesting to note that the search for a collective benefit such as minimizing power losses drives the grid to a solution that demands feasible amounts of reactive power injection/absorption (and currents). This is not the case of the intermediate results (Figure 5) where the numbers for reactive power and currents may dramatically stress several of the grid inverters. In fact it may imply violating standards related to the power factor or imposing currents out of the inverters operative capacity.

Finally, it can be observed that the final reactive power profile (bottom sub-figure of Figure 6) provides design guidelines for LV grids (when the lines are mainly resistive). It indicates that both voltage regulation and power loss minimization is achieved when those inverters that see more impedance (nodes 23 and 24) do the job, that is, they absorb reactive power. Taking this guideline to the extreme could mean that by using only one highly-rated static synchronous compensator (STATCOM, [30]) at the point of the grid where more impedance is seen could be enough for meeting both control demands. The analysis of this strategy is left for future work.

D. Discussion

One of the standard uses of reactive power control methods is to cope with the over-voltage problem by making use of the power capacity of the distributed inverters. The study carried out in this paper indicates that the optimal settings for $Q_i$ at each inverter overcomes the over-voltage problem while offering an additional feature such as power loss minimization. However, achieving the collective benefit may be delicate and several issues may need further attention.

First of all, it should be deeper studied whether different types of changing conditions in the grid may provoke a drastic change in the optimal reactive power profile. This analysis should include different changes in the grid topology, connection and disconnection of loads, daily/monthly/yearly active power profiles, or changes in the infrastructure such as type of lines (resistive versus inductive). If changing conditions determine different reactive power profiles, an online implementation of the optimization approach should be required,
which may impose the deployment of ICT and the possibly adoption of a master/slave configuration. The master could receive all the active power $P_i$ that each inverter is injecting. Then, the master would determine the optimal $Q_i^*$ to be sent to all inverters. And depending on the computational overhead of the implemented algorithm in the master node, two different approaches are foreseen. When the computational overhead is low, the optimal $Q_i^*$ could be computed in the master node and send to each inverter whenever required. When the computational overhead is high, the optimal $Q_i$ for each inverter should be computed offline according to the production profiles and other predicted events, and stored in the master acting then as a central database. In system operation, $Q_i^*$ updates could be central whenever required.

V. CONCLUSION

This paper has presented a study focusing on the optimal reactive power settings for distributed inverters in PV-LV grids aimed at minimizing system power losses. The study has been performed using simulation tools on a real-inspired 24 inverters grid set-up. The results indicate that the problem formulation and its solution permits regulating voltages as well as minimizing power losses by imposing a feasible reactive power setting to the distributed inverters. Future work will analyze whether the obtained results still hold in a more realistic setup where implementation aspects will also be considered.

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